

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/14881816>

The Perception of Movement and Depth in Moiré Patterns

Article in *Perception* · February 1993

DOI: 10.1068/p220287 · Source: PubMed

CITATIONS

35

READS

2,064

1 author:



[Lothar Spillmann](#)

University of Freiburg

194 PUBLICATIONS 4,340 CITATIONS

SEE PROFILE

The perception of movement and depth in moiré patterns

Lothar Spillmann

Neurologische Universitätsklinik mit Abteilung für Neurophysiologie, Hansastrasse 9,
D-7800 Freiburg i. Br., Germany

Received 18 November 1991, in revised form 12 March 1992

Abstract. Moiré patterns can produce striking movement effects and in more complex stimuli can induce vivid stereoscopic depth. The physical rules underlying these phenomena are reviewed and their relationship to psychophysics is discussed.

First, it is shown how moirés in 'optical line interference' patterns are created by superimposing periodic visual stimuli, eg gratings, and shifting them relative to each other. When two gratings are presented in this manner, small differences in spatial frequency, orientation, and speed are magnified. This magnification has prompted the use of moiré patterns both in industry and in art where their enhanced sensitivity to misalignment and spatial distortion has been widely exploited.

Next, it is demonstrated how enhanced depth in 'stereoscopic interference' patterns is produced by presenting grating stimuli in two (or more) depth planes. The perceived depth effect in the resulting moiré pattern can be elicited similarly by binocular disparity and motion parallax.

Finally, it is described how perceived movements occurring in different directions and at different depths are the basis for the perceptual 'irritations' that fascinate observers in complex moiré patterns. The use of moirés for the noninvasive examination of the human retina by aliasing is discussed.

1 Introduction

Moiré patterns, readily observed in undulating lace window curtains and often seen in op-art, have been known for many centuries. First produced in textiles and tapestries, they probably originated in China (fragments found in Northern Mongolia were dated about the 10th century) and found their way via Persia to central Europe (Ackerman 1967; Oster 1968). The term 'moiré' comes from the French and derives from moiré silk or 'watered' silk which is produced from a glossy fabric with a fine ribbed weave of parallel cords. The technique requires that the fabric be dampened and folded with the face side inward and with the two selvages running together side by side. The cloth is then passed between heated steam rollers under considerable pressure. By this procedure part of the ribs are flattened while others are left in relief, the flattened and raised parts in the unfolded fabric producing an irregular, glistening pattern of curiously waved dark and bright bands. These bands result entirely from the varying angles at which the rays of light are reflected from the surface of the fabric. [For an example see Burnham (1981), page 178.]

Similar phenomena have been reported for other stimuli. Roget (1825) described moiré patterns produced by the spokes of a carriage wheel passing in front of a vertical railing, and Faraday (1831) extended this observation to the sectors of two cardboard disks presented in counterrotation. The next to study moirés was Lord Rayleigh (1874) who laid down the mathematical foundations of moiré fringes perceived on finely ruled, superimposed, diffraction gratings (parallel equispaced lines etched onto a polished glass surface to produce optical spectra by diffraction). Presumably, moiré fringes were a common feature also of steel engravings, which often used superimposed sets of narrow parallel lines to vary contrast (courtesy of Dr N Wade). In this century, Guild (1956, 1960) developed the use of moiré patterns for measuring minute displacements; and Oster and Nishijima (1963), Nishijima and

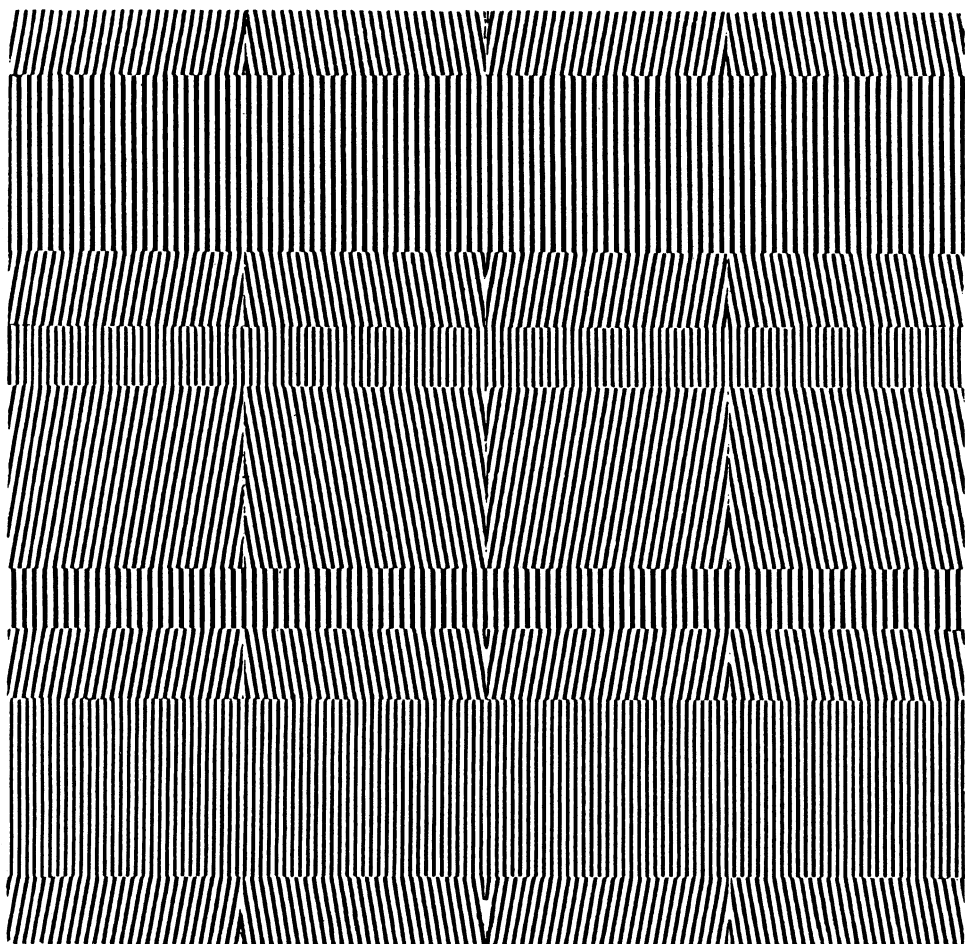


Figure 1. A complex moiré pattern consisting of horizontal and vertical gratings is obtained when foil I (transparency supplied with this issue) is aligned with and superimposed onto the printed pattern. Sliding the foil in small circles across the pattern will produce enhanced multidirectional moiré motion. When the foil is held about 4 cm above the pattern, slight changes of head position will elicit similar movements by motion parallax. Recommended viewing distance is 1 m. (Multiple SCHB I/1986; courtesy of Ludwig Wilding.)

Oster (1964), and Oster (1968) suggested moiré patterns as tools for the optical examination of surface structures in the material sciences (eg experimental strain analysis during mechanical distortion and thermal expansion). Oster et al (1964) showed that moiré patterns may also serve to illustrate the spatial distribution of equipotentials around an electric dipole.

From these studies it appears that most, if not all, of the moiré effects can be discussed merely in terms of physics, without regard to visual perceptions and illusions (eg Kafri and Glatt 1990). However, although this may be true, the phenomenological implications of formal equations to vision scientists and artists are far from clear, and thus a quantitative treatment of moiré patterns and their dependence on parametric stimulus variations is warranted (eg Wade 1978). Such treatment is particularly needed to resolve the apparent discrepancy between the stimulus and the percept. Observers of moiré patterns continue to be fascinated by the everchanging moiré fringes occurring in moving grating displays. They are perplexed by the fact that a small shift or tilt of the head relative to the moiré stimulus results in disproportionately large displacements and rotations. And they are puzzled why two gratings spaced only a few inches apart will lead to the impression of striking three-dimensional depth.

In this article I seek to resolve this discrepancy by emphasising that the perception of the moiré fringes is based not on the properties of the individual gratings, but on the luminance modulations resulting from their superposition. In the moiré, small differences between spatial frequencies (ie the numbers of cycles per degree of visual angle) of the component gratings are magnified. The same applies to differences in grating orientation, speed of motion, and to perceived depth, all of which are greatly enhanced. These effects are not illusory. They are the consequence of (physical) luminance modulations in the composite stimulus itself.

The article begins with a general description of moiré patterns and their practical applications (section 2). The two main perceptual aspects are then illustrated—enhanced motion and depth (sections 3 and 4), and their combination in complex patterns (section 5). Finally, the use of moiré patterns as a psychophysical test in the noninvasive study of the retinal receptor mosaic is discussed (section 6).

Figure 1 shows an example of a complex moiré from Ludwig Wilding, the noted artist and professor at the Kunsthochschule in Hamburg. On placing foil I over this pattern and aligning the two, one perceives rows of horizontal and vertical gratings having similar spatial frequencies. If now the foil is moved about, the gratings will appear to move at different speeds in four different directions: left, right, up, and down. When the foil is held some 4 cm above the pattern, such as to create a fixed interspace, all these motions can be seen with the slightest translation of the head, owing to motion parallax (see section 4.3). Note that with increasing separation between the foil and the pattern the moiré fringes change their orientation from horizontal to diagonal until at a distance of approximately 20 cm they disappear. Note also that any rotation of the foil relative to the printed pattern of figure 1 results in large spatial frequency differences. How are these effects to be explained? Let us consider the individual perceptual properties of moirés one by one.

2 General properties of moiré patterns

The term moiré pattern refers to the patterns that are produced when two (or more) geometrically periodic structures, such as parallel screens (sheer curtain fabric, wire mesh in window screens), or gratings (fences, railings), are superimposed. Alternating dark and bright moiré bands are then perceived where the two overlapping patterns pass in and out of alignment, producing an 'optical interference' (Wilding 1973). Such interference is a well-known problem in printing, when a halftone picture (consisting

of a regular array of tiny dots) is screened a second time. The two halftone screens interfere with each other and spurious lines and shades arise which are not present in the original. Another common example is seen when a fabric with finely spaced near-horizontal lines is shown on television. Here, disturbing moiré patterns result from the interaction of the lines with the raster of the television.

The basis for this interference is the difference in the spatial frequencies (and orientations) of the superimposed patterns. A freeway overpass provides a good example. The vertical railings of the front and rear fences have the same periodicity everywhere. However owing to the different viewing distances, the near fence subtends a larger visual angle than the far fence. It thus has a lower spatial frequency at the eye, and so a moiré arises because of the continuous change of phase between the railings. As one approaches the overpass, the moiré stripes in the periphery appear to start 'running' towards the centre. At the same time they get finer and ultimately disappear when the relative difference in distance between the near and far fences, and therefore the difference between spatial frequencies, exceeds a critical value. When one moves away from the overpass, the moiré reappears in the opposite sequence (from fine to coarse) and 'running' in the opposite direction.

2.1 *Spatial interference, sampling, and beating*

Periodic patterns can be combined in two different ways, and both ways produce 'optical interferences' which are in many ways similar. In the illustrations used in this paper the patterns are combined by multiplication: the light perceived at any given point of the moiré is the product of the amount of light which the background grating reflects and the proportion of reflected light which the superimposed grating allows to pass. If one or the other pattern is black, ie passes or reflects no light, one sees only black; otherwise one sees white. A dark stripe is perceived where the two different gratings are in counterphase—when either the black bars of the top grating occlude the white spaces of the bottom grating; or, conversely, when the transparent spaces on top allow the black grating bars underneath to be seen. In comparison, a light stripe is perceived where the two gratings are in register. Typically, the dark moiré stripe looks narrower than the light moiré stripe, but both are much wider (have a lower spatial frequency) than the individual bars of the component gratings (Oster and Nishijima 1963).

One can also think of this method of combination as a sampling procedure. That is, one cannot see the entire background pattern; rather, only periodic patches or samples are revealed through the clear spaces in the top grating (undersampling). The moiré pattern that is perceived is a kind of distortion called aliasing which under-sampling often produces. The topic of aliasing is discussed further in section 6.1.

Periodic patterns can also be combined by addition, so that the light present at each point of the moiré is the sum of the light present in each of the two individual patterns. As a consequence two stripes added in counterphase will produce a stripe of medium brightness (in contrast to darkness in the preceding case). An example of additive combination results when each pattern is projected onto a screen with a separate slide projector, and the two images are superimposed and rotated relative to each other. This procedure also results in moiré patterns, although of lower contrast (Bryngdahl 1976; Lim et al 1989).

It is not clear why a moiré pattern can be seen under these conditions. A linear addition of two sinusoidal frequencies contains only the two component frequencies and, in particular, contains no sinusoid corresponding to the moiré spatial frequency (Burton 1973). The fact that we see the stripes must therefore be attributed to some kind of nonlinearity within the visual system (Burton 1973; Bryngdahl 1976). One example of a nonlinearity that could cause moirés to appear is the logarithmic

luminance transformation of the stimulus within the photoreceptors. Additional nonlinearities may be introduced by the functional properties of receptive fields at various levels of the visual system. These and other transformations may explain why additive moirés are perceived although their frequency components are not contained in the original stimulus. Burton (1973) has shown that the contrast sensitivity curve for moiré gratings generated in this manner is similar both in shape and in peak position to the contrast sensitivity data for single sine-wave stimuli whose spatial frequencies are matched to the respective difference frequency.

Spatial interference in moirés must not be confused with the physical interference produced by waves of coherent light interacting on the retina, ie ocular laser interferometry (Campbell and Green 1965). Unlike optical interference fringes, moiré fringes involve the interaction between intensity distributions on a much larger scale than the wavelength of light. However, whereas multiplicative intensity distributions already exist in the extraocular stimulus before entering the eye (Oster and Nishijima 1963; Wilding 1975), additive moiré fringes first arise at the level of the retina and can be observed only if we are able to resolve the original grating stripes. When a pattern giving rise to an additive moiré is scanned across with a photometer having a sufficiently large aperture, the luminance is constant. Thus, perception of the bright and dark moiré stripes must be based on a nonlinear processing of the contrast modulation inherent in the composite stimulus.

2.2 Superposition

The dark and bright moiré bands that originate when two gratings are overlaid correspond to the width and spacing in the spatial beat pattern. As a result of this superposition, one perceives a 'new' grating having either a sinusoidal (for sine-wave gratings) or a triangular (for square-wave gratings) luminance distribution and a spatial frequency that is lower than the spatial frequencies of the component gratings. This can be demonstrated in figure 2. When foil I is statically superimposed onto this figure, the resulting moiré grating is much coarser than both the gratings on the foil and the background underneath. Oster and Nishijima (1963) called this enhancement of differences between the component gratings the *magnification effect*. This effect is a key feature of moirés; it applies not only to spatial-frequency differences, but also to orientation, speed of motion, and depth. The enhancement is analogous to the effect used when a string instrument is tuned with a tuning fork: the beats 'amplify' the frequency differences.

As with acoustic beats produced by the pulsation of two tones of similar frequencies, optical interference between grating stimuli is termed 'spatial beating' in the psychophysical literature (Burton 1973). Visual beats occur when luminance gratings of slightly different frequencies are superimposed (Oster 1968; Bryngdahl 1981; Badcock and Derrington 1985, 1987; Derrington and Badcock 1986). The beats are perceived wherever adjacent luminance maxima or minima are spatially fused. In analogy with music, the new spatial frequency of the moiré grating is equal to the difference between the spatial frequencies of the two original gratings (Bryngdahl 1974, 1975, 1976). Since in figure 2 the absolute frequency difference between the background and the foil is approximately the same for both halves of the background grating, the resulting moiré grating looks uniform all across.

When foil I is raised above figure 2, the shorter distance to the eye progressively lowers the spatial frequency of the grating on the retina. In the right half, this reduction enlarges the difference between the spatial frequencies of the gratings. As a consequence the moiré grating becomes finer and finer until it can no longer be seen. In comparison, in the left half the difference between spatial frequencies first decreases, ie the moiré grating becomes coarser, then passes through zero, ie there is

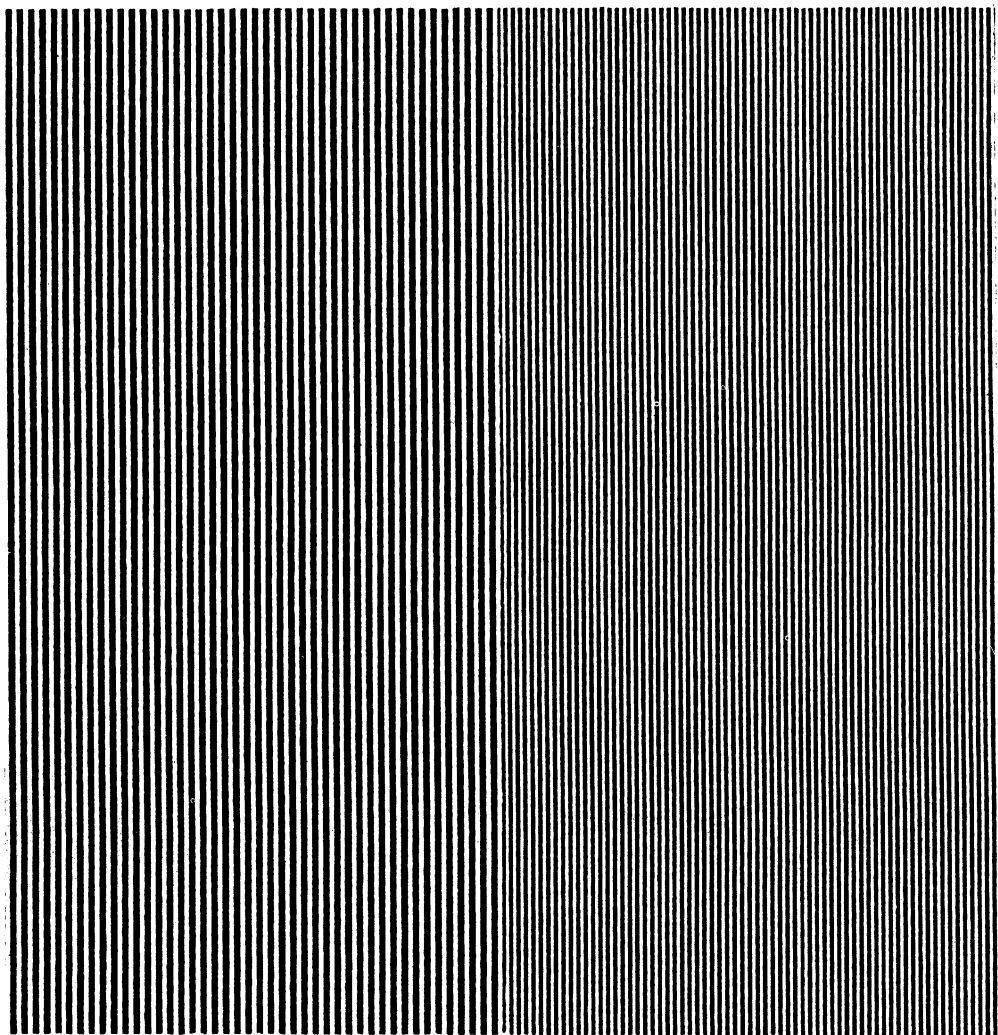


Figure 2. A static moiré grating of uniform appearance is produced by aligning and superimposing foil I (supplied with this issue) onto figure 2. Shifting the foil in a horizontal direction will result in two gratings moving equally in opposite directions. Slight rotation of the foil will cause these gratings to appear tilted away from each other like a 'fishbone' pattern. Recommended viewing distance 1 m. (Courtesy of Ludwig Wilding 1976.)

no moiré, and thereafter increases again, but now with the opposite sign. As a result the moiré reoccurs until it again disappears.

The smaller the difference between the spatial frequencies of the gratings, the lower the spatial frequency of the moiré and the larger the magnification. For example, with component frequencies of 29 and 30 cycles deg^{-1} , and thus a moiré frequency of 1 cycle deg^{-1} , a magnification factor of 30 can readily be attained. However, with higher spatial frequencies, such as in Ronchi rulings, much larger magnification factors are also possible. [For a quantitative analysis of the relationship between the spatial frequencies of the component gratings and the resulting spatial frequency of the moiré see equation (A1) in the Appendix.]

Since the difference frequency between the component gratings determines the spatial frequency of the moiré grating, it follows that a finite number of frequency pairs should yield the same percept as long as the difference between each pair of frequencies is the same. This is indeed the case. However, two high-spatial-frequency gratings will produce a more homogeneous moiré pattern than two low-spatial-frequency gratings. This points towards a critical ratio, in the formation of moirés, between the difference frequency and the spatial frequencies of the component gratings. When the ratio is too large as is the case for two coarse gratings, a kind of undersampling of one grating by the other takes place resulting in a moiré that is spotty and not representative. The fact that moiré fringes of superior quality are perceived when the component gratings are blurred by defocusing, ie removal of high-frequency 'noise', or when the spatial frequencies are too high to be resolved by the human eye [Lord Rayleigh (1874); see also section 6.1], is the best evidence that it is not the individual gratings to which our eye responds, but the resulting beat frequency.

2.3 *Practical applications*

Several practical uses of moiré patterns in high-precision measurement derive from the magnification of spatial-frequency differences. One important application in this regard is quality control testing of optical flats and lenses (Oster and Nishijima 1963). By placing the lens between two identical gratings, deviations from geometry can be readily observed. Any small distortion of the first grating pattern, caused by its passage through the lens, will lead to strong and reproducible irregularities in the resulting moiré pattern. In this way small changes of refractive index become easily visible. The same magnification makes it possible to see dislocations in the structure of a crystal lattice that amount to only 0.1 nm or deviations of angle corresponding to $1/3600$ degree (Oster 1968). By virtue of these properties moirés can also be applied to detect infidelities between cheap replicas of ruled diffraction gratings and the master grating from which they were made. Further use may be found in facilitating the reading of vernier scales on calipers, micrometers, micromanipulators, and computer disk head positioners, where movements, in the micrometer range, are easily determined with a photoelectric cell by counting the associated moiré fringe displacements (for reviews see Oster 1965a; Luxmoore and Shepherd 1983; Gasvik 1987).

For similar reasons moiré patterns are used in biology to detect microorganisms suspended in fluids (Nishijima and Oster 1964). The basic assumption here is that every frequency (average density) of these microorganisms produces its own physical beat pattern; and that it is thus possible to use optical moirés as an inexpensive, but highly sensitive, 'lens-free' microscope. Owing to the enormous magnification factor inherent in such patterns, small perturbations will cause large perceptual distortions in the resulting moiré. This is why very high accuracy (0.1 mm) is required in the manufacture of patterns to be used for producing moirés (Wilding 1975). Any deviations from the rule will be greatly enhanced especially when one of the gratings is moved.

3 Perceived motion

3.1 Linear translation

Even more conspicuous than the dark and bright stripes in static moirés are their movements in dynamic moirés. With each translation of the foil over figure 2, the moiré bands are seen to move simultaneously in one or the other direction. As a consequence, the difference in spatial frequency between the two halves of the background grating is immediately revealed. Motion occurs in the same direction if the spatial frequency of the moving grating is higher than the spatial frequency of the stationary grating (left). Motion in the opposite direction results when the relationship between spatial frequencies is reversed (right). Thus, when the spatial frequencies of the gratings, as well as the direction of the shift, are known, the direction of the movement can be readily predicted (Bryngdahl 1975; Wilding 1977; Kafri and Glatt 1990).

Right and left movement may be explained by the displacement of the moiré pattern as illustrated in figure 3. In the upper two lines of each triplet, the bars of the two component gratings generating the moiré pattern are represented by short dashes of slightly different lengths. Note that the bars are uniform in width and spacing corresponding to square-wave gratings with a duty cycle of 1. On the other hand, the bar width on the moiré pattern (line 3) varies continuously and the spacing between bars varies inversely to their width. This change of bar width results from the change of relative phase between the two component gratings, and is similar to the maxima and minima of a sine-wave grating varying in luminance (Kondo et al 1990). However, this similarity is a result of undersampling (see section 2.2) and is attributable to the choice of low spatial frequencies. If gratings with higher spatial frequencies are used, the bar width varies linearly, which indicates that the luminance distribution in the difference (moiré) grating is, in fact, triangular.

If in figure 3 the grating with the higher spatial frequency (line 2 on the left) is displaced relative to the grating having the lower spatial frequency (line 1 on the left), the moiré grating will appear to move in the same direction as the shift (to the right as indicated by the arrow). Conversely, if the grating with the lower spatial frequency (line 2 on the right) is displaced relative to the grating having the higher spatial frequency (line 1 on the right), the moiré grating will appear to move in the opposite direction to the shift. Note that the amplitude of the displacement (lines 3) in either case is considerably greater than the amplitude of the shift of the grating (lines 2). It is because of this fact that observers can detect the shift of a moving grating superimposed onto a stationary grating even when the shift itself is too small (or its speed too slow) to be detected alone (Badcock and Derrington 1985). Thus, another important feature of moiré patterns is *displacement and velocity magnification*.

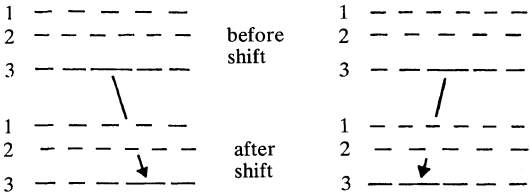


Figure 3. Schematic explanation of perceived movement in a moiré pattern (line 3) elicited when one grating (line 2) is shifted rightward relative to another grating (line 1). Depending on whether the moving grating has a higher or lower spatial frequency than the stationary grating, the moiré appears to move in the same (left) or opposite direction (right) as the component grating. This is indicated by the two arrows depicting the direction of displacement.

In analogy to the moiré bands themselves, their perceived movement is based on the changing distribution of the space-averaged luminance within the composite stimulus. The moiré motion therefore also is real, not apparent (illusory). This may seem surprising in view of the fact that the perceived speed and amplitude of the motion are much greater, and more compelling, than the small displacement of the component grating by which the motion is induced (Oster and Nishijima 1963). However, note again that the visual stimulus responsible for the enhanced motion is the beat speed of the moiré pattern, not the shifts of the individual gratings.

This velocity magnification can be readily tested by partially superimposing foil I onto figure 2. When this is done, the actual velocity and amplitude of the displacement of the foil outside the area of overlap can be directly compared with the perceived moiré motion within. As a rule, the higher the spatial frequencies of the two component gratings and the smaller their difference, the greater is the speed of the moiré movement. In finely ruled grating patterns (Wilding 1973), this relationship results in large flashes of brightness and darkness traversing the pattern at 'lightning' speed and eliciting the perception of metallic gloss and vibration. Effects such as these prompted Canaday (1965) to describe moiré patterns as art that "pulses, quivers and fascinates". [For a quantitative analysis of the relationship between the displacement and speed of the component grating(s) on one hand and the displacement and speed of the moiré on the other see equations (A2) and (A3) in the Appendix.]

3.2 Rotation

If the grating on foil I is slightly rotated relative to the grating bars in figure 2, the result is an inclined moiré pattern. However, as with motion direction, the direction of tilt is different for the two grating halves. On the left, where the spatial frequency of the foil is higher than the spatial frequency of the background grating, the moiré bands rotate in the same direction as the foil; on the right, where the frequency is lower, the bands rotate in the opposite direction.

Note that a small inclination of only a few degrees will cause a large change in the perceived tilt of the moiré pattern. For example, in figure 2, a 5° tilt of one of the component gratings produces about four times this tilt in the resulting moiré grating. This feature is called *orientation magnification* (Oster and Nishijima 1963), and can be tested by analogy to velocity magnification by using partial overlap between foil and background. The precise value of the tilt factor depends on the difference in the spatial frequency of the gratings (the smaller the spatial-frequency difference, the higher the magnification). When the two spatial frequencies are the same, the orientation of the moiré grating is perpendicular to the bisecting line between the two component gratings.

Figure 4 (from Wilding 1973) illustrates the effect of stimulus rotation on the orientation and width of the moiré bands in the case of (a) two intersecting lines (comparable to a pair of telegraph wires or scissor blades); and (b) two intersecting gratings of equal spatial frequencies. When the angle of intersection in (a) is large (left), the wedge-shaped dark areas between the two lines are relatively short. In comparison, when the angle of intersection is small (right), the dark areas extend over a much greater length. As a consequence, the fringe spacing in (b) increases from narrow to wide as the angle of intersection is reduced. At the same time the inclination of the moiré bands relative to the component gratings approaches 90° (see also Oster and Nishijima 1963). This example shows how a change of angle between two gratings affects not only the orientation of the moiré, but also its spatial frequency. Lord Rayleigh (1874) gave a mathematical description of this relationship, which has been exploited by Wade (1974) in a study of the perceptual effects obtained with rotating gratings. Moiré patterns are thus not only very precise

indicators of lateral displacement or phase mismatch in gratings having different spatial frequencies (section 2.2), they are also extremely sensitive indicators of angular misalignment in gratings having different orientations.

The second pattern in figure 4b reveals the moiré microstructure. Here it can be seen that the moiré bands are formed by a series of small rhombs produced by the intersecting grating bars. The centre of the bright fringe coincides with the short diagonal in each of the white rhombs. In comparison, the dark fringe is centred on the region of greatest density of the black bars (Oster and Nishijima 1963; Bryngdahl 1976; Wade 1978). The luminance profiles both of the bright and of the dark fringes are triangular; however, the profile of the dark fringe is clipped at the trough. This may be the reason why the dark fringe looks narrower and more conspicuous than the bright fringe.

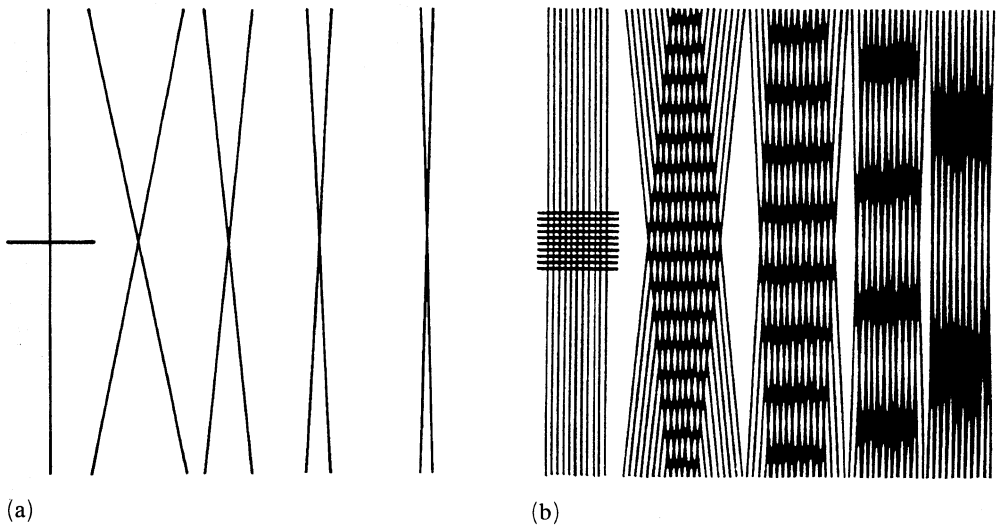


Figure 4. (a) The spatial beat pattern between two lines intersecting each other at angles of 90°, 24°, 12°, 6°, and 3° (from left to right). Note the change in length of the wedge-shaped darkenings accompanying the intersection. (b) Moiré stripes produced by two gratings intersecting each other at the same angles. The spatial frequency of the moiré pattern decreases with the decrease in intersection angle. (From Wilding 1973.)

3.3 *Random displacement*

Besides using one-dimensional gratings, there are other ways of producing moiré patterns. For example, one may superimpose, and displace, curved, instead of linear, grating stimuli; or combine two-dimensional gratings (chequerboards). Still other possibilities are to superimpose, and displace, concentric circles, radial ray patterns, and wavy patterns. In this way, perception of rotating stars, wheels, and corrugated surfaces may be elicited. The books by Vasarely (1969), Wilding (1973, 1987), and Wade (1982) contain excellent illustrations of the moiré patterns produced in this manner.

Figure 5 shows moiré-type phenomena in MacKay's (1957a, 1957b) well-known 'concentric ring' and 'radial ray' patterns. Note that in the ring pattern bright, radial 'fans' may be seen to oscillate perpendicularly to the direction of the contours. Analogously, in the ray figure, a circular shimmer may be perceived to revolve approximately at right angles to the radiating lines. These phenomena have been interpreted as phantom moirés caused by a positive afterimage (a weak persisting trace of the original stimulus) superimposed onto the inducing pattern. If such an

afterimage were misaligned by small eye movements, it could act as a second pattern to produce the observed brightness fluctuations (MacKay 1957b; Oster and Nishijima 1963; Oster 1968). However, this hypothesis has been challenged, and an alternative explanation based on meridional blur from transient astigmatism has been proposed instead [see Wade (1978) for a discussion of this issue.]

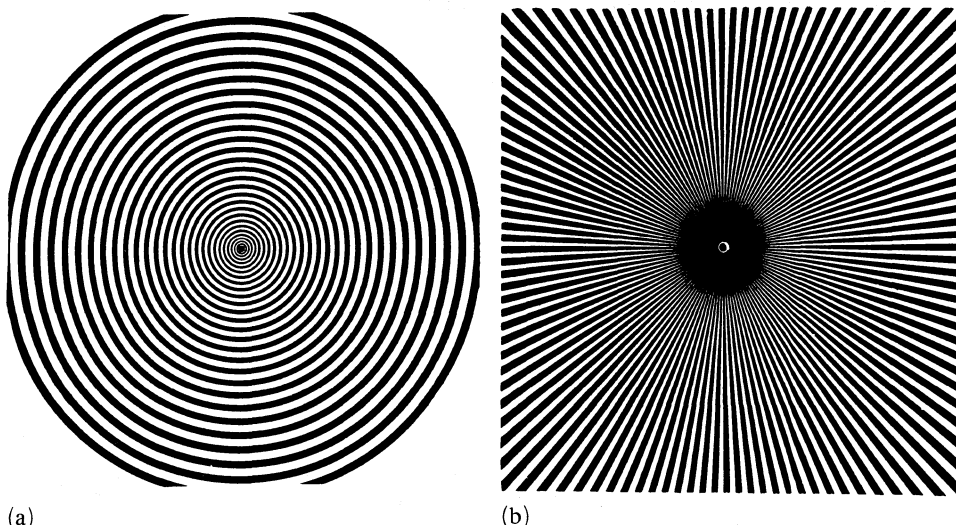


Figure 5. (a) Concentric ring pattern: revolving fans are seen to move propeller-like around the centre. (b) Radiating ray pattern: shimmering lines may be perceived which are approximately concentric with the centre. (From MacKay 1957a, 1957b.)

4 Apparent depth

Moiré patterns fascinate not only by creating greatly enhanced motion, they also create vivid depth. Apparent depth and perspective from texture gradients are well-known effects in visual science (Gibson 1950), and have also been exploited in the pseudoreliefs of op-art (eg Vasarely 1969; Riley 1978; Wilding 1979; Wade 1982). On such contoured surfaces (eg a series of dampened sine-waves as in Riley's "Current"), corrugations in depth are strongly suggested (Rogers and Graham 1979). However, much stronger effects can be achieved with real reliefs such as a folded grating serving as a background for producing moirés. In these patterns, binocular disparity (the small differences between the images formed in the right eye and the left eye) combines with motion parallax to elicit the perception of enhanced depth which is strikingly real, even superrealistic.

4.1 Disparity from phase shift

The synergistic interaction between disparity and motion parallax is exploited in the stereoscopic multiples of Ludwig Wilding. To create depth in moiré patterns, none of the usual devices for right and left image separation such as mirror or prism stereoscopes, crossed polarizers, red-green glasses, engraved prism grooves, and phase-different haploscopes are needed. Instead, Wilding (1977) employs two (or more) depth planes with slightly different spacings (and sometimes orientations) for foreground and background gratings. In addition, he typically uses a grid of thick black reference lines drawn on the background to provide a framework for binocular fusion.

This technique of producing depth from superimposed grating stimuli has been called 'stereoscopic interference' or 'occlusion stereoscopy' (Wilding 1975, 1977, 1979, 1987). It is based on the disparate moiré patterns for the two eyes resulting

from the difference in spatial frequency between the front and rear gratings (Tyler 1973; Wade 1982; Kondo et al 1990). Note that, in analogy to movement and rotation, disparity refers to the moiré patterns themselves rather than the component gratings by which they are produced (and which are largely invisible). Wilding (1982) has shown that when the moiré bands are presented as red and green anaglyphs and viewed through spectacles with green and red filters (Julesz 1971), the same depth effect is perceived as with superimposed black-and-white gratings. Typically, the same initial latency for emerging depth is also found in both. The perception of depth in moiré patterns is hardly surprising since the disparity signal for the moiré fringes exists for any binocular cell which integrates luminance over the width of the grating bars.

Figure 6 [adopted from Kondo et al (1990); for comparison see Chiang (1967)] illustrates schematically how depth of different polarity may be produced by the appropriate choice of grating spatial frequencies. First consider the case where the spatial frequency of the foreground grating (m) is slightly higher than that of the background grating (n). When the observer fixates binocularly on one of the reference lines (B) on the background grating, the retinal image for each eye can be simulated by projecting the foreground grating (A) on the background grating while retaining the same spatial frequency. This projection is equivalent to shifting the grating bars rightward on the background for the left eye (to A_L) and leftward for the right eye (to A_R). This shift produces crossed disparity because projecting the higher-spatial-frequency grating onto the lower-spatial-frequency grating implies that the moiré pattern formed by the two gratings would move in the same direction as the grating bars (according to figure 3, left). This crossed disparity is a cue to the visual brain that the moiré pattern lies in front of the fixation plane.

Now consider the case where the spatial frequency in the foreground is lower than that on the background ($m < n$). Again, the moiré patterns for each eye can be simulated by projecting the foreground grating on the background grating. This projection is also equivalent to shifting the grating bars rightward on the background

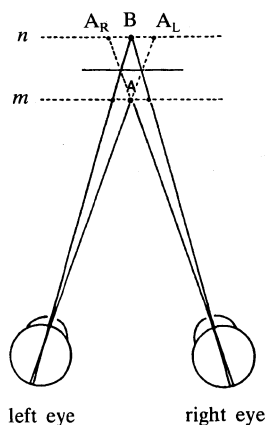


Figure 6. Schematic representation of foreground (A) and background (B) grating as viewed by the left eye and the right eye. When B is fixated, the images corresponding to A on the two retinas can be simulated by projecting A onto the fixation plane (A_L and A_R). When the spatial frequency of the foreground grating (m) is higher than that of the background grating (n), the monocular moiré patterns arising from the superposition will move in the same direction as the half images (ie away from the fixation point). The resulting crossed disparity is a cue to the visual brain indicating that the pattern is located before the fixation plane. When m is lower than n , the monocular moiré patterns will move in the opposite direction, resulting in uncrossed disparity and an apparent location behind the fixation plane. (Adopted from Kondo et al 1990.)

for the left eye and leftward for the right eye. However, in this case the resulting moiré pattern shifts in the opposite direction to the grating bars (according to figure 3, right). This shift produces uncrossed moiré disparity and, thus, a moiré pattern that appears to lie behind the fixation plane. [For a quantitative analysis of the relationship between grating disparity and moiré disparity see equation (A4) in the Appendix.]

Note that positive and negative disparities in moiré patterns are obtained by fixating on the same background plane. The difference in sign results from the reversed relationship between the spatial frequencies of the front and rear gratings producing opposite shifts of the moiré pattern [in agreement with equation (A2) in the Appendix].

4.2 Disparity from tilt

The enhanced depth caused by disparate moiré patterns is readily visible if the background is folded into a relief. For example, if one folds the background grating shown in figure 7 into a double-V-shaped profile (see legend for details) and then superimposes foil II onto it, two sharply angled, wedge-shaped moiré gratings are perceived, one pointing forward (top) and the other backward (bottom). The wedges completely depart from the plane of the folded paper. This striking feature of moiré patterns is called *depth magnification*.

The reason for the enhanced depth lies in the combination of spatial-frequency difference (as outlined in figure 6) and the difference in angle of orientation (due to the relief). Both cause displacements in opposite directions and orientations which the moiré then magnifies (in accordance with figure 2). Because of the V-shaped folds, the flat projection of the background grating no longer is parallel, but for each slope resembles a spatial-frequency gradient either from coarse to fine (receding) or from fine to coarse (approaching). When the two opposite progressions are overlaid with a uniform foreground grating, they produce monocular moiré patterns characterised by large lateral shifts and angular tilts, thereby greatly enhancing disparity. These displacements can be seen by first closing one eye, then the other. Whereas in the upper half of figure 7 the monocular moiré pattern jumps in the opposite direction to the movement of the eye (crossed disparity), in the lower half it jumps in the same direction (uncrossed disparity). The monocular patterns resemble series of open angles pointing either outward ($\langle \dots \rangle$ top) or inward ($\rangle \dots \langle$ bottom). [For illustrations see Oster (1968), his figures 37 and 38; and the front cover of Wilding (1982).]

Note that the perceived depth of these stimuli varies with observation distance in an asymmetrical manner. Whereas the forward pointing wedge becomes slightly steeper as the observer approaches the figure, the backward pointing wedge becomes shallower. Note further that if the background profile in figure 7 is reversed, the perceived depth polarity of the moiré pattern reverses accordingly.

The magnification in depth is illustrated, for a stepped background, by the apparent relief (dashed line) in figure 8. In this context Wilding (1977) uses the term 'perspective' to denote the enhanced depth produced by the converging lines. However, as shown by figure 7, the perceived depth is monocularly ambiguous since it can correspond either to a projecting or to a receding edge. This is not what is typically meant by perspective.

Moiré patterns appearing to bulge out of or recede beyond the reference frame by as much as 1.5 m or more are common among Wilding's works, which sometimes measure in excess of 1 m in height and in width. In fact, the perception of folds, steps, and cylinders produced by his background grating reliefs is so vivid that observers spontaneously explore the perceived depth with their own hands. This great depth probably has prompted the use of stereomoirés in advertising (Kondo 1983).

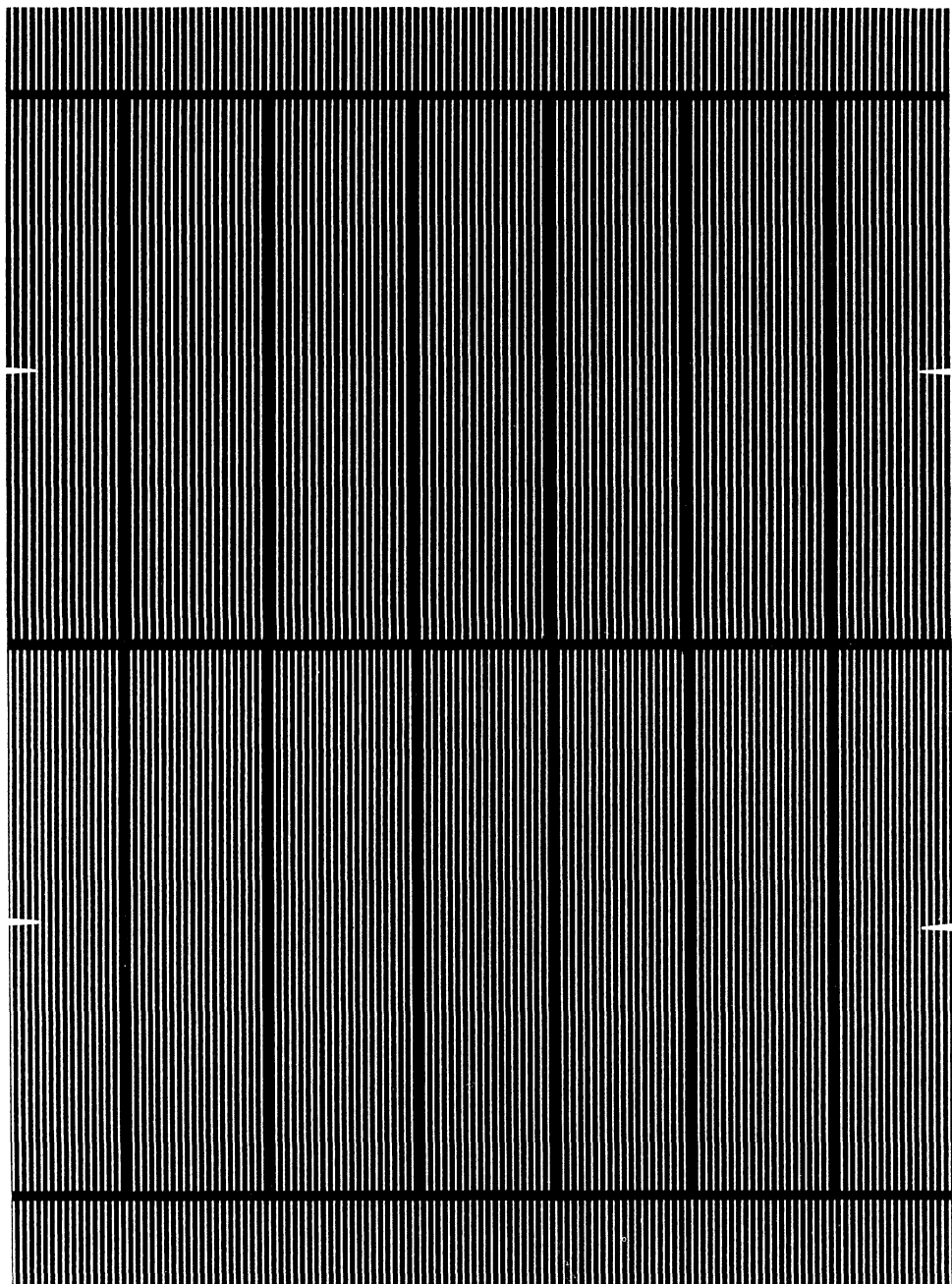


Figure 7. To perceive enhanced depth, take the exact copy of this figure supplied with this issue and produce a background relief conforming to a shallow double-V-shaped profile. This is done by first folding the background in the middle along the heavy black line. The fold should be forward, forming an angle of about 30° . Thereafter fold the upper and lower background halves each along a horizontal line connecting the left and right arrows. Both folds should be backward resembling shallow Vs (of about 150° angle). Finally, bend the upper and lower edges of the background backward, so that the entire concertina-like profile supports itself when resting on a horizontal surface. Foil II (supplied) should now be collinearly aligned with and accurately superimposed onto the resulting relief. Make sure that the foil is plane and supported by the three ridges. After a few seconds a pair of sharply angled moiré wedges will emerge, the upper one pointing forward, the lower one backward. Note that strong depth can also be perceived with one eye if combined with slight lateral movement of the head. Recommended viewing distance 1 m. (Multiple CPSR 5/1986; courtesy of Ludwig Wilding.)

Since the calibration of disparity is dependent upon convergence, viewing a large moiré pattern from afar will lead to a greater perceived depth than will viewing a smaller one with the same disparity from close-up. [For a quantitative analysis of the relationship between grating depth and moiré depth see equation (A5) in the Appendix.]

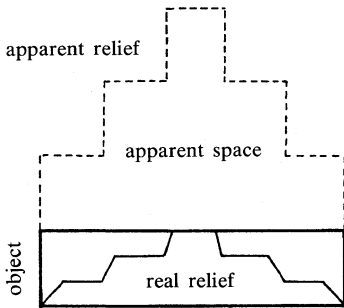


Figure 8. Top view of the box (heavy rectangle) for presenting a flat foreground grating against a folded grating background relief. The foreground grating is located immediately behind the front Plexiglas cover of the box (at bottom). This stimulus arrangement elicits perception of a relief that appears much deeper (dashed line) than it actually is. (From Wilding 1977.)

4.3 Motion parallax

So far, all effects described have been assumed to be binocular in origin. It is therefore not surprising that looking at figure 7 with one eye only reduces the perceived depth significantly. However, this is only true if both the stimulus and the observer are stationary. Strong, undiminished, depth effects are immediately reinstated if the observer moves relative to the grating patterns. Motion parallax caused by the different angular velocities for foreground and background [Wheatstone (1838); see Wade (1987) for historical origins] is evidently as powerful a cue as lateral disparity for perceiving depth in moiré patterns. Also, it does not have an equivalent for binocular rivalry which will occur at close observation distances when disparities are too large to be fused.

Note that both wedges may be seen even from an extreme lateral angle, although their relative positions in space change systematically with each change in the location of the observer. Thus, when the observer moves to the right, the forward pointing wedge appears to move to the left, while the backward pointing wedge appears to move to the right; when the observer moves up, the forward pointing wedge likewise moves up, while the backward pointing wedge moves down. These motion-parallax shifts are appropriate to the perceived depth of the wedges as defined by lateral disparity (Tyler 1974), and they are similarly magnified by the moiré. As with depth from binocular parallax, this is probably the reason why the perceived shape of the wedges does not remain constant. As one moves around, the wedges appear to stretch and contract as though suffering from plastic deformation. When the stimulus pattern is rotated by 90° , perceived depth is greatly reduced if not absent because the images received by the right eye and the left eye no longer produce lateral disparity.

The equivalence of motion parallax and disparity as cues for depth is supported by experiments showing that the gain of parallax motion elicited by head movements can be adjusted to produce any amount of depth, and that parallax and stereoscopic depth effects are comparable in strength [Rogers and Graham (1979, 1982); for a quantitative analysis of the relationship between grating motion parallax and moiré parallax see equation (A6) in the Appendix].

In summary: The moiré in figure 7 magnifies as well as reverses the disparity and motion parallax of the two shallow Vs of the background thereby leading to the perception of wedges having opposite polarity. The greatly enhanced depth in both stimuli originates from the magnified displacements (shifts and tilts) of the monocular moiré patterns, thus producing much larger disparities than would be expected from the shallow relief of the background.

An interesting way of deriving depth information in the absence of disparity or motion parallax has been described by Takasaki (1970, 1973). He employs moiré fringes to represent the topography of a three-dimensional object. This is done by using collimated light to cast the shadow lines of a high-spatial-frequency grating onto the object and then viewing (or photographing) it through the same grating, although from a different angle [see Eschbach and Bryngdahl (1983), their figure 1]. When the distance between the grating and the object is small and the distance between the camera and the grating is large, one obtains moiré contour lines which perfectly model the object in depth. Figure 9 illustrates this effect for a female torso. These contour lines are comparable to the isoelevation lines used in topographical maps. In medicine moiré depth contouring has been applied to the analysis and reconstruction of anatomical shapes, notably in patients suffering from deformities of the spine (scoliosis) or the cornea (keratotonus) (van Wijk 1980).

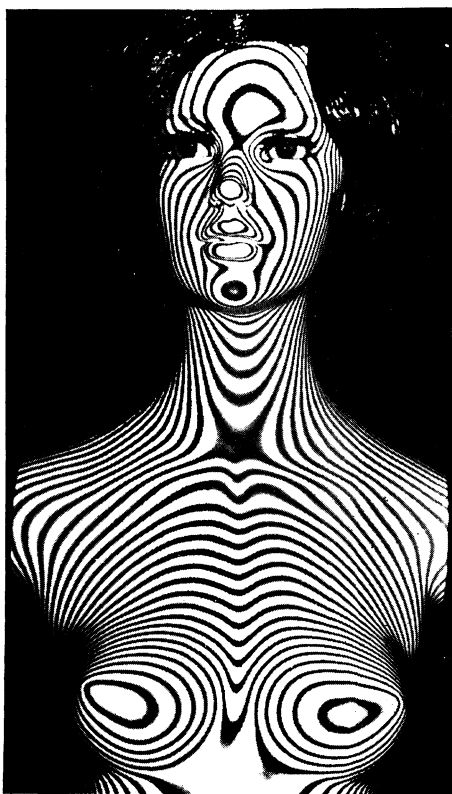


Figure 9. Moiré lines of a mannequin produced by projecting a vertical grating onto the torso and photographing the resulting contours through the same grating, but from a different angle. The moiré pattern arises from the superposition of the parallel grating lines onto the deformed shadow lines. Note that the perceived depth in this figure is not diminished by closing one eye. (From Takasaki 1970.)

5 The puzzle of complex moiré patterns

Why are moiré patterns so fascinating? There are five reasons, all related to the perceptual magnification of small stimulus differences: first, the perceived spatial frequency of a moiré pattern is much lower than the spatial frequencies of each of the component gratings (see section 2); second, the perceived speed of a moiré pattern is much higher than the actual velocity of the grating shift (section 3); third, the perceived change in orientation of a moiré pattern is much more pronounced than the physical rotation inducing it (section 3); fourth, the perceived depth of a moiré pattern vastly exceeds the depth of the folded background relief (section 4); and fifth, the perceived motion parallax and depth inherent in complex moiré patterns work synergistically (section 4).

In addition to these properties, moiré grating patterns are perplexing by demonstrating that the observers may by their own head and body movements simultaneously elicit perception of stimulus movements that have different directions and speeds, even though the two inducing gratings are stationary. This flow and counterflow in incoherent moiré motion patterns cannot be reconciled with the observer's body movements (Ueberschaar 1988), nor can it be attributed to the individual grating patterns themselves. Figure 1 shows how the multidirectionality is produced by a patchwork of separately manipulated gratings differing in relative spatial frequency and orientation. When this grating patchwork is seen through a uniform foreground grating (foil I), four kinds of opposing movements are perceived (in accordance with section 3).

To some observers, the multiple movements in figure 1 give the impression that they occur in different planes and thereby determine the apparent segmentation of the figure into different surfaces. Grating patches with the same direction of movement tend to be perceived as belonging to the same depth plane (ie being at the same level), whereas grating patches moving in different directions tend to be located at different depth planes (stacked). An explanation of this apparent stratification may come from the Gestalt principles of 'common fate' and 'completion'. According to these principles, a depth plane emerges when spatially separated, 'incomplete', stimuli behave similarly (ie have the same spatial frequency and angular orientation and move in the same direction), suggesting that the 'missing' parts between them can be accounted for by occlusion. Often, such surfaces are delineated by sharp illusory contours (Kanizsa 1976) separating one apparent depth plane from the other.

Wilding's complex moirés are designed to combine both enhanced movement and enhanced depth for a more scintillating perceptual effect. One cannot escape these strong phenomena even if one attempts to assume a passive attitude. Occasionally, the stimulus pattern challenges the observer to the point where it becomes agitating and nauseating. Thus, observers have compared the percepts elicited by stereokinetic op-art works to those experienced during migraine attacks (Wade 1978). It is likely that both are accompanied by massive neuronal discharges in early visual centres.

6 Moiré patterns as psychophysical tools

6.1 Aliasing

Recently moiré patterns have become an important noninvasive tool for the study of human retinal anatomy. With the use of 'aliasing', a technique known from signal analysis in imaging systems, the distribution of the retinal cones can be determined in the living eye without the need for preparing any histological sections (Williams and Collier 1983; Yellott 1983; Williams 1985, 1988; Coletta and Williams 1987; Coletta et al 1990). Aliasing occurs when a grating of very high spatial frequency (above 50–60 cycles deg^{-1}) is presented to the observer's eye. Because of the limited spatial resolution of the eye, the observer will not perceive the grating itself, but

instead will see an irregular, 'zebra-striped', pattern of low spatial frequency. This second 'grating' is actually a moiré pattern which results from the superposition of the high-spatial-frequency grating onto the cone receptor mosaic of the retina (Oster 1965b; Ohzu et al 1972).

Moirés occur only if the distribution of the photoreceptors is regular (Yellott 1982), as indeed it is in the fovea (Hirsch and Hylton 1984). Thus, the second 'grating' revealed by the moiré is the observer's own mosaic of retinal receptors. Anatomical studies of the density and size of the photoreceptors are consistent with the results obtained with aliasing. If a high-spatial-frequency grating is projected onto a histological section of the fovea, the same zebra-striped moiré pattern may be perceived as that previously reported by the observers participating in the psychophysical experiment (Williams 1985).

6.2 *Dichoptic viewing*

Moiré patterns may be perceived with two eyes as well as with one eye (figure 1). One may therefore ask whether they are also present when one grating is presented to the left eye and the other to the right eye, ie in dichoptic viewing? To test this question a phase-difference haploscope (developed by Professor E Aulhorn, Tübingen) was used. This apparatus allows the two component gratings to be presented dichoptically without binocular rivalry. The gratings were slightly tilted relative to each other and were projected through two sector disks spinning at 60 Hz in counterphase (episcotister principle). They were observed one with each eye through a correlated pair of counterrotating disks thus affording a combined view of the gratings. Under these conditions moiré bands are completely absent.

This result is consistent with earlier observations by Oster (1965b, 1968), who found moiré lines to be present in afterimages of crossed gratings generated monocularly, but not dichoptically; and by Kaufman (1974), who reported seeing no fans in MacKay's concentric ring and radial ray patterns (figure 5) when they were shown interocularly (see, however, Piggins 1978). It also agrees with findings by Badcock and Derrington (1987) according to which the displacement threshold for a grating improved (because of the resulting moiré) when a second grating was presented to the same eye, but deteriorated somewhat when it was presented to the other eye. With dichoptic stimulation spatial beats were never perceived. One must therefore assume that each monocular stimulus is processed independently and that the spatial interference required for the formation of moirés cannot be achieved by the interaction between binocular channels in the visual system. On the other hand, weak dichoptic moirés have been described for additive zone-plate structures (Bryngdahl 1976). Furthermore, depth moirés resembling a series of horizontally segregated layers can be elicited by presenting tilted grating stimuli stereoscopically (Piggins 1978; Tyler 1980). But these phenomena are not strictly comparable to the bright and dark bands occurring in luminance moirés.

6.3 *Coloured moirés*

Despite the fact that high-contrast black-and-white gratings produce the strongest moiré patterns, it would also be worthwhile exploring the use of coloured gratings and backgrounds (Oster 1968; Bryngdahl 1981). A coloured moiré effect in everyday life may be observed when a bridge spanning a river is approached from one of the river banks. When the posts of the railings are painted in different colours and are seen against a coloured background, such as a green meadow or a yellow field, a coloured moiré grating will be observed. Like its black-and-white counterpart (see the example of the freeway overpass, section 2), this coloured moiré grating originates externally to the eye. It would be interesting to find out whether the movement disappears under conditions of isoluminance, where motion perception has been shown to break

down (Ramachandran and Gregory 1978; Cavanagh et al 1984). When instead of solid objects coloured foils are used in conjunction with coloured gratings, moiré fringes form by virtue of subtraction (Armstrong 1982). This is yet another way of producing moiré patterns (Bryngdahl 1976, 1981).

7 Résumé

At a time when empirical researchers are highly specialised and interested only in a narrow field, the combined study of vision by artists, psychophysicists, and engineers is a welcome challenge. It teaches us that the perception of motion and depth in moiré patterns, although perplexing, is not an illusion. Both can be understood and explained in terms of the moiré patterns themselves rather than the component stimuli from which they arise. On the other hand, careful psychophysical experiments designed to test, for example, the exact correspondence between moiré disparity and perceived depth have yet to be done. It is therefore hoped that this paper will stimulate such empirical enquiries into the quantitative aspect of perception when moiré patterns are viewed.

Acknowledgements. I thank Professor L Wilding, Hamburg, for making the multiples described in this paper available and J Obergfell for helping in the early stages of this research. I also thank Drs J I Nelson, N J Wade, M Kondo, D R Williams, J P Thomas, A Jedynak, and Th Meigen for extensive discussions, and Drs W Ehrenstein, M Bach, H Höfler, and L O Harvey for comments on an earlier draft. The help of Mr Th Strohgriemler in preparing the foils and backgrounds is gratefully acknowledged. This work was supported by the German Research Council (SFB 325, B4).

References

- Ackerman P, 1967 "Textiles of the Islamic periods, A. History" in *A Survey of Persian Art* second printing, volume V, chapter 52, Eds A U Pope, P Ackerman (Oxford: Oxford University Press) pp 1995–1996
- Armstrong T, 1982 *Make Moving Patterns* (Stradbroke, England: Tarquin)
- Badcock D R, Derrington A M, 1985 "Detecting the displacement of periodic patterns" *Vision Research* **25** 1253–1258
- Badcock D R, Derrington A M, 1987 "Detecting the displacements of spatial beats: A monocular capability" *Vision Research* **27** 793–797
- Bryngdahl O, 1974 "Moiré: Formation and interpretation" *Journal of the Optical Society of America* **64** 1287–1294
- Bryngdahl O, 1975 "Moiré and higher grating harmonics" *Journal of the Optical Society of America* **65** 685–694
- Bryngdahl O, 1976 "Characteristics of superposed patterns in optics" *Journal of the Optical Society of America* **66** 87–94
- Bryngdahl O, 1981 "Beat pattern selection—multi-color-grating moiré" *Optics Communications* **39** 127–131
- Burnham D K, 1981 *A Textile Terminology* (London: Routledge and Kegan Paul)
- Burton G J, 1973 "Evidence for non-linear response processes in the human visual system from measurements on the thresholds of spatial beat frequencies" *Vision Research* **13** 1211–1225
- Campbell F W, Green D G, 1965 "Optical and retinal factors affecting visual resolution" *Journal of Physiology (London)* **181** 576–593
- Canaday J, 1965 "Art that pulses, quivers and fascinates" *The New York Times Magazine* (21 February)
- Cavanagh P, Tyler C W, Eizner Favreau O, 1984 "Perceived velocity of moving chromatic gratings" *Journal of the Optical Society of America A* **1** 893–899
- Chiang C, 1967 "Stereoscopic moiré patterns" *Journal of the Optical Society of America* **57** 1088–1090
- Coletta N J, Williams D R, 1987 "Psychological estimates of extrafoveal cone spacing" *Journal of the Optical Society of America A* **4** 1503–1513
- Coletta N J, Williams D R, Tania C L M, 1990 "Consequences of spatial sampling for human motion perception" *Vision Research* **30** 1631–1648
- Derrington A M, Badcock D R, 1986 "Detection of spatial beats—nonlinearity of contrast-increment detection?" *Vision Research* **26** 343–348

- Eschbach R, Bryngdahl O, 1983 "Subcoded information carriers: Hybrid moiré system" *Journal of the Optical Society of America* **73** 1123–1129
- Faraday M, 1831 "On a peculiar class of optical deceptions" *Journal of the Royal Institution* **1** 205–223
- Gasvik K J, 1987 "Moiré technique" in *Optical Metrology* chapter 5 (New York: John Wiley) pp 117–144
- Gibson J J, 1950 *The Perception of the Visual World* (Boston, MA: Houghton Mifflin)
- Guild J, 1956 *The Interference Systems of Crossed Diffraction Gratings* (Oxford: Clarendon Press)
- Guild J, 1960 *Diffraction Gratings as Measuring Scales* (London: Oxford University Press)
- Hirsch J, Hylton R, 1984 "Quality of the primate photoreceptor lattice and limits of spatial vision" *Vision Research* **24** 347–355
- Julesz B, 1971 *Foundations of Cyclopean Perception* (Chicago, IL: University of Chicago Press)
- Kafri O, Glatt I, 1990 *The Physics of Moiré Metrology* (New York: John Wiley)
- Kanizsa G, 1976 "Subjective contours" *Scientific American* **234**(4) 48–52
- Kaufman L, 1974 *Sight and Mind* (Oxford: Oxford University Press)
- Kondo M, 1983 "Stereomoiré as advertising media" *Annual Reports, Yoshida Zaidan* **16** 317–326
- Kondo M, Wade N J, Nakamizo S, 1990 "Geometrical analysis of the motion and depth seen in moiré patterns" *Bulletin of the Faculty of Literature, Kitakyushu University* **22** 97–114
- Lim J S, Kim J, Chung M S, 1989 "Additive type moiré with computer image processing" *Applied Optics* **28** 2677–2680
- Luxmoore A R, Shepherd A T, 1983 "Applications of the moiré effect" in *Optical Transducers and Techniques in Engineering Measurement* chapter 3, Ed. A R Luxmoore (London: Applied Science Publishers) pp 61–108
- MacKay D M, 1957a "Moving visual images by regular stationary patterns" *Nature (London)* **180** 849–850
- MacKay D M, 1957b "Some further visual phenomena associated with regular patterned stimulation" *Nature (London)* **180** 1145–1146
- Nishijima Y, Oster G, 1964 "Moiré patterns: Their application to refractive index and refractive index gradient measurements" *Journal of the Optical Society of America* **54** 1–5
- Ohzu H, Enoch J M, O'Hair J C, 1972 "Optical modulation by the isolated retina and retinal receptors" *Vision Research* **12** 231–244
- Oster G, 1965a "Moiré optics: A bibliography" *Journal of the Optical Society of America* **55** 1329–1330
- Oster G, 1965b "Optical art" *Applied Optics* **4** 1359–1369
- Oster G, 1968 *The Science of Moiré Patterns* 2nd edition (Barrington, NJ: Edmund Scientific)
- Oster G, Nishijima Y, 1963 "Moiré patterns" *Scientific American* **208**(5) 54–63
- Oster G, Wasserman M, Zwerling C, 1964 "Theoretical interpretations of moiré patterns" *Journal of the Optical Society of America* **54** 169–175
- Piggins D, 1978 "Moirés maintained internally by binocular vision" *Perception* **7** 679–691
- Ramachandran V S, Gregory R L, 1978 "Does colour provide an input to human motion perception?" *Nature (London)* **275** 55–56
- Rayleigh, Lord, 1874 "On the manufacture and theory of diffraction gratings" *Philosophical Magazine* **47** 81–93
- Riley B, 1978 *Works 1959–78* (London: The British Council)
- Rogers B J, Graham M E, 1979 "Motion parallax as an independent cue for depth perception" *Perception* **8** 125–134
- Rogers B J, Graham M E, 1982 "Similarities between motion parallax and stereopsis in human depth perception" *Vision Research* **22** 261–270
- Roget P M, 1825 "Explanation of an optical deception in the appearance of the spokes of a wheel seen through vertical apertures" *Philosophical Transactions of the Royal Society* **115** 131–140
- Takasaki H, 1970 "Moiré topography" *Applied Optics* **9** 1457–1472
- Takasaki H, 1973 "Moiré topography" *Applied Optics* **12** 845–850
- Tyler C W, 1973 "Depth perception in disparity gratings" *Nature (London)* **251** 140–142
- Tyler C W, 1974 "Induced stereomovement" *Vision Research* **14** 609–613
- Tyler C W, 1980 "Binocular moiré fringes and the vertical horopter" *Perception* **9** 475–478
- Ueberschaar G, 1988 "Moiré—ein optisches Phänomen und seine Anwendung" *Deutsche Optiker-zeitung* 26–36
- van Wijk M C, 1980 "Moiré contourgraph—an accuracy analysis" *Journal of Biomechanics* **13** 605–613
- Vasarely V, 1969 *Vasarely* [volume 1 of the German edition, translated by H G Schürmann] (Neuchâtel, Switzerland: Griffon)

-
- Wade N J, 1974 "Some perceptual effects generated by rotating gratings" *Perception* **3** 169–184
- Wade N J, 1978 "Op art and visual perception" *Perception* **7** 21–46
- Wade N J, 1982 *The Art and Science of Visual Illusions* (London: Routledge and Kegan Paul)
- Wade N J, 1987 "On the late invention of the stereoscope" *Perception* **16** 785–818
- Wheatstone C, 1838 "Contributions to the physiology of vision—Part the first. On some remarkable, and hitherto unobserved, phenomena of binocular vision" *Philosophical Transactions of the Royal Society* **128** 371–394
- Wilding L, 1973 *Räumliche Irritationen: Optische Interferenzen perspektivischer Täuschungen* (Köln, Kölnischer Kunstverein)
- Wilding L, 1975 *Sehen und Wahrnehmen: Scheinbewegungen, räumliche Irritationen, stereoskopische Scheinräume* (München, Galerie Klihm)
- Wilding L, 1976 *Stereoskopische Scheinräume, Objekte und Anaglyphen* (Düsseldorf, Städtische Kunsthalle)
- Wilding L, 1977 *Sehen und Wahrnehmen: Untersuchungen und Experimente* (Buchholz/Hamburg: Dr. Johanns Knauel)
- Wilding L, 1979 *Visuelle Autobiographie* (Düsseldorf: Galerie Schoeller)
- Wilding L, 1982 *Bilder für zwei Augen* (Düsseldorf: Galerie Schoeller)
- Wilding L, 1987 *Retrospektive 1949–1987* (Kaiserslautern: Pfalzalerie des Bezirksverbands Pfalz)
- Williams D R, 1985 "Aliasing in human foveal vision" *Vision Research* **25** 195–205
- Williams D R, 1988 "Topography of the foveal cone mosaic in the living human eye" *Vision Research* **28** 433–454
- Williams D R, Collier R, 1983 "Consequences of spatial sampling by a human photoreceptor mosaic" *Science* **221** 385–387
- Yellott J I, Jr, 1982 "Spectral analysis of spatial sampling by photoreceptors: Topological disorder prevents aliasing" *Vision Research* **22** 1205–1210
- Yellott J I, Jr, 1983 "Spectral consequences of photoreceptor sampling in the rhesus retina" *Science* **221** 382–385

APPENDIX: Equations governing moiré patterns (from Kondo et al 1990)

(1) Spatial frequency of moiré (M) as a function of the spatial frequencies of the moving and stationary component gratings (m and n , respectively):

$$M = m - n. \quad (\text{A1})$$

(2) Displacement of moiré (S) as a function of the displacement of the moving grating (S_x):

$$S = \frac{m}{m - n} S_x. \quad (\text{A2})$$

If $m > n$, moiré moves in the same direction as the moving grating; if $m < n$, it moves in the opposite direction.

(3) Velocity of the moiré (V) as a function of the velocity of the grating (V_x):

$$V = \frac{m}{m - n} V_x. \quad (\text{A3})$$

(4) Disparity produced by moiré (M_δ) as a function of the disparity produced by the component gratings (δ):

$$M_\delta = \frac{m}{m - n} \delta, \quad (\text{A4})$$

where m and n are the spatial frequencies of the foreground and background gratings, respectively.

If $m > n$, M_δ is positive, crossed disparity, pattern appears in front;

if $m < n$, M_δ is negative, uncrossed disparity, pattern appears in back.

(5) Depth of moiré (M_d) as a function of the separation between component gratings (d):

$$M_d = \frac{m}{m - n} d. \quad (\text{A5})$$

(6) Motion parallax produced by moiré pattern (M_θ) as a function of the motion parallax produced by the component gratings (θ):

$$M_\theta = \frac{m}{m - n} \theta. \quad (\text{A6})$$